## Assignment 8 Problem Reduction to Maximum Network Flow Problem

In this assignment your grade will not only depend on the correctness of your solution but on the clarity of your explanations. That includes clear labeling and neatness of diagrams, completeness and conciseness of explanation, etc. I strongly urge you to have someone who will make good suggestions for improvement to review your answers before you submit them.

**Problem 1: Testing Base Station Coverage**

A town is planning on where to place the base stations for a new wireless technology. It wants to test the coverage of a set of **k base station locations** before it goes to the expense of building them. The base station positions, B1, B2, …, BK , will be specified by Bi(x, y), the x and y coordinates in the plane of base station Bi. The town wants you to design an algorithm that will test how well the base stations serve N mobile clients in a specific scenario. The scenario is consist n mobile clients, C1, C2, …, CN positioned in the town at coordinates Ci(x, y).

Each client will be connected to exactly one base station. However, the base stations have limited range. A client can only be connected to a base station within distance R – distance can be thought of as the usual Euclidean distance in the x, y coordinates. Finally, there is a load parameter L – no more than L clients can be connected to any single base station.

Specify a flow network to solve this problem including any data structures. You must use the notation in the problem and draw a picture so the representation is easy for a human to understand!

1. Specify what each node, edges, and edge capacities represent in the network.
2. How to determine which mobile clients are assigned to which base stations for a given scenario.

## Problem 2: IT department holiday scheduling

Congratulations! You have been promoted to be manager of IT organization of your favorite company. However every position has its headaches. Since it is imperative that the companies systems be available every day of the year, you must ensure that there is a (exactly one) supervisor at the data center every day of the year including all the holidays. **Holidays** are single days off that most employees prefer not to work including days like Thursday, Friday, Saturday, and Sunday of Thanksgiving week, every day during Christmas week, etc.

Some holidays occur during **holiday periods**. For example the **Thanksgiving holiday period** might be **Thursday, Friday, Saturday, and Sunday of Thanksgiving week** for your company. For the purposes of this problem we will assume there are k holiday periods. Some of these will have a single holiday but many will have more than one day**, e.g. Thanksgiving holiday period has 4 holidays**. Let **Dj** be the number of holidays in period **j**.

Luckily, there are sufficient supervisors to cover all these days even with the following constraints.

1. There are exactly **n** supervisors, each supervisor provides a list of **exactly k** of the holidays they are willing to work. Exactly one supervisor should be assigned to each holiday and that holiday must be on that supervisors list.
2. No supervisor should be assigned to cover more than **C** holiday days in a year. (assume that k >C)
3. No supervisor should work more than one holiday in each of the holiday periods.

You could write an algorithm to **determine if** there is an assignment of supervisors to holidays that satisfies the above constraints where the supervisors only work on the days from their list - BUT **you cleverly remember that the maximum flow algorithm your learned at Cal Poly is good at solving matching problems.**  Thus you decide to try to model the problem as a maximum flow problem. Then you can just use the shortest augmenting path algorithm to solve the problem!

1. First do the problem **without constraint #3**. That is, the supervisors may work more than one day in a holiday period. Doing the problem means determining a flow network that models the problem.   
   G = (V, E, u) and draw a picture of it. **Use labels for the vertices and edges that represent the different things you are trying to model, namely**

**Supervisors = { S1 , S2 , …. , Sn }  
Holidays = { H1 , H2 , …. , Hm }** where m is the total number of holidays.

**Holiday Periods = {HP1 , HP2 , …. , HPk } (note that the sum of the Dj is m )**

Any symbols you use must be clearly defined before the diagram.

1. **Then develop the model to solve the problem with all three constraints.** Again determine a flow network that models the problem. G = (V, E, u) and draw a picture of it. Any symbols you use must be clearly defined before the diagram.

**For each diagram (A and B)**

1. Explain why the maximum flow in the flow network solves the problem in a short paragraph. Namely why does the maximum flow obtained represent a solution to the problem; Satisfies the contraints etc.
2. Explain how to tell if there is no solution to the problem given the list of holidays that each supervisor is willing to work.
3. Explain how to tell what holidays are assigned to what supervisor.